The Shape–Slope Relation in Observed Gamma Raindrop Size Distributions: Statistical Error or Useful Information?

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ABSTRACT

The three-parameter gamma distribution $n(D) = N_0D^\mu \exp(-\Lambda D)$ is often used to characterize a raindrop size distribution (DSD). The parameters μ and Λ correspond to the shape and slope of the DSD. If μ and Λ are related to one another, as recent disdrometer measurements suggest, the gamma DSD model is simplified, which facilitates retrieval of rain parameters from remote measurements. It is important to determine whether the μ–Λ relation arises from errors in estimated DSD moments, or from natural rain processes, or from a combination of both statistical error and rain physics.

In this paper, the error propagation from moment estimators to rain DSD parameter estimators is studied. The standard errors and correlation coefficient are derived through systematic error analysis. Using numerical simulations, errors in estimated DSD parameters are quantified. The analysis shows that errors in moment estimators do cause correlations among the estimated DSD parameters and cause a linear relation between estimators $\hat{\mu}$ and $\hat{\Lambda}$. However, the slope and intercept of the error-induced relation depend on the expected values μ and Λ, and it differs from the μ–Λ relation derived from disdrometer measurements. Further, the mean values of the DSD parameter estimators are unbiased. Consequently, the derived μ–Λ relation is believed to contain useful information in that it describes the mean behavior of the DSD parameters and reflects a characteristic of actual raindrop size distributions. The μ–Λ relation improves retrievals of rain parameters from a pair of remote measurements such as reflectivity and differential reflectivity or attenuation, and it reduces the bias and standard error in retrieved rain parameters.

1. Introduction

Accurate characterization of raindrop size distribution (DSD) and the estimation of DSD parameters using remote measurements are needed for inferring rain microphysics. Because various factors contribute to the formation and evolution of rain DSDs, a single explicit functional form has not been found. Hence, simple functions have been used to model a rain DSD.

Historically, an exponential distribution with two parameters was used to characterize rain DSD. Special cases of exponential DSDs were determined by Marshall and Palmer (1948) and Laws and Parsons (1943). However, subsequent DSD measurements have shown that the exponential distribution does not capture "instantaneous" rain DSDs and a more general function is necessary.

Ulbrich (1983) suggested the use of the gamma distribution for representing rain DSD as

$$n(D) = N_0D^\mu \exp(-\Lambda D). \tag{1}$$

The gamma DSD with three parameters (N_0, μ, and Λ) is capable of describing a broader range of raindrop size distributions than an exponential distribution (a special case of the gamma distribution with $\mu = 0$). The three parameters of the gamma DSD can be obtained from three estimated moments. It was shown that the three parameters are not mutually independent (Ulbrich 1983; Chandrasekar and Bringi 1987; Kozu 1991; Haddad et al. 1997). Hence attempts were made to derive rain

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DSDs from a set of mutually independent parameters (Haddad et al. 1997). However, correlations among DSD parameters, if real, may be useful in reducing the number of unknowns and enable the retrieval of the DSD from a pair of independent remote measurements such as reflectivity and attenuation, as in the case of the dual-wavelength radar technique, or the reflectivities at horizontal and vertical polarization, as in the case of polarimetric radar. An approach to find relationship among the DSD parameters had been proposed by Kozu et al. (1999). An N_0–μ relation was found and used by Ulbrich (1983) for retrieving the three DSD parameters from reflectivity and attenuation. The derived relation was later attributed to statistical error (Chandrasekar and Bringi 1987) and is unstable depending on the method of fitting procedure. Moreover, fluctuations in N_0–μ range over several orders of magnitude; hence, the utility of the relation is limited.

5. Summary and discussion

In this paper, detailed analyses of error propagation from moment estimators to the estimated gamma DSD parameters were performed. A mathematical approach was used to quantify the effects of errors in moments on DSD parameters and on R and D_0 retrievals. The retrievals using the μ–Λ relation were compared with the fixed μ approach. The μ–Λ relation is believed to capture a mean physical characteristic of raindrop spectra and is useful for retrieving unbiased DSD parameters when only two independent remote measurements are available such as Z and Z_{DR} or attenuation.

Theoretical analyses and numerical simulations confirm that errors in moment measurements (estimates) can cause high correlations in gamma DSD parameters such as that observed between $\hat{\mu}$ and $\hat{\Lambda}$ for a single pair of expected values in Fig. 5. This error effect, however, should not be equated to the μ–Λ relation (2) derived from a quality-controlled dataset of rain DSD measurements. The slope and intercept of the linear relation associated with moment error depend on the particular values of μ and Λ, whereas the mean values of retrieved DSD and physical parameters are not biased by fluctuation errors in the moments. The moment errors have little effect on the μ–Λ relation for rain rates that contribute most to rain accumulations. The μ–Λ relation is consistent with observation whereby heavy rain is represented with large drops having a broad distribution. Compared to the gamma distribution with a fixed μ, the constrained gamma distribution with the μ–Λ relation is more flexible in representing a wide range of instantaneous DSD shapes.

Recognizing the difficulty of separating statistical errors and physical variations, we believe the errors in DSD parameter estimators should not be considered meaningless; rather they should be studied further for the following reasons.

1) The errors in the estimated DSD parameters are linked to the functional relations between DSD parameters and moments. The correlations among the estimated gamma DSD parameters due to moment errors are a result of DSD fitting (moment method), and a requirement of unbiased moments and physical parameters.
2) Natural rain DSD may not be the same as the mathematically modeled gamma distribution. In the model we have used here, the difference between actual DSD and assumed gamma distribution can be attributed to errors in the moment estimators.
3) ‘‘Fluctuation’’ is a more appropriate description than ‘‘error’’ in characterizing the differences of DSD parameters or moments from their expected values since each realization could be a real physical event. The DSD parameters should be allowed to vary as in Zhang et al. (2002). It is very difficult to separate the physical variations from statistical errors.
4) Nevertheless, measurements always contain errors and as a result the correlation between $\hat{\mu}$ and $\hat{\Lambda}$ may be strengthened. If such a correlation can improve retrievals such that the bias and standard error in physical parameters are minimized, it can be a valuable addition to the retrieval process.

It has been shown that rain DSD retrievals from radar measurements that use the μ–Λ relation agree with the in situ measurements better than those obtained with fixed μ. The relation is thought to capture the physical nature of rain DSDs and should provide a way to improve DSD estimation by dual-parameter radar retrieval techniques.

We derived the μ–Λ relation (2) from video disdrometer measurements in Florida during the summer of 1998 for moderate and heavy rain case ($R > 5$ mm h^{-1}) to minimize the sampling error effect. The relation (2) should be extendable to rain rates smaller than 5 mm h^{-1}. The relation is also valid for the observations collected in Oklahoma. It is possible that the μ–Λ relation changes depending on climatology and rain type. If that is true, a tuned μ–Λ relation based on local DSD observations should be derived and used for accurate rain DSD estimation.